

系所(組)別：材料科學與奈米科技研究所

考試科目：工程數學

1. (40%) Solve $y(x)$ for the following ordinary differential equation:

(a) $y'' + 5y' + 6y = 0$ (10%)

(b) $y'' - 16y = e^{4x}$ (10%)

(c) $x^2 y'' - 3xy' + 4y = 0$, with $y(1) = 2$ and $y'(1) = 8$ (10%)

(d) $y' - y - xy^5 = 0$, with $y(0) = 1$ (10%) Hint: use $u = 1/y^4$

2. (15%) Derive the Laplace transform for the function $\{\sinh kt\}$ is

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2 - k^2} \text{ with } s > k$$

3. (20%) The eigenvalues of the matrix $M = \begin{bmatrix} a & -a & 0 \\ -b & 2b & -b \\ 0 & -a & a \end{bmatrix}$ are 0, 1 and 7.

(a) Find the values of a and b (assume: $a > 0$ and $b > 0$) (10%)

(b) Find the normalized eigenvectors for matrix M . (10%)

4. (25%) Using the Fourier series to expand the following function:

(a) $f(x) = x^2$, with $-\pi < x \leq \pi$ (15%) (b) if $x = \pi$, please show that: $\sum_{n=1}^{\infty} \frac{1}{n^2} = ?$ (10%)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \text{ with } -L < x < L$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, \quad n = 0, 1, 2, 3, \dots \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, \quad n = 1, 2, 3, \dots$$

$$\int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax + \left(\frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax + C$$

$$\int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3} \right) \sin ax + C$$