1. (50%) Solve y(x) for the following differential equations.

(a) y''-8y'+16y=0 with y(0)=3 and y'(0)=6 (15%)

(b) y''+3y'+2y=20sin x with y(0)=0 and y'(0)=-6 (20%)

(c) xy'+y^2=1 with y(1)=3 (15%)

2. (20%) Solve the following differential equation by Laplace transform only.

$\frac{d^4y}{dx^4} = A, \textit{with} \quad y(0) = y''(0) = y(L) = y''(L) = 0 \quad \textit{and } 0 \le y \le L$ **用其他方法不予計分**

3. (20%) matrix M= $\begin{pmatrix} \alpha & 0 & 3\beta \\ 0 & \alpha & 4\beta \\ 3\beta & 4\beta & \alpha \end{pmatrix}$**, the three eigenvalues of the matrix M are**

λ_1, λ_2, λ_3 and λ_1:λ_2:λ_3=7:2:-3, and det(M)=-84. Please find the α and β

[assume α>0 and β>0]

4. (10%) f(x,y,z)=xyz exp(-αz^2), find the gradient of f (∇f) . [where α>0

and $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ **]**