

1. (50%) Solve $y(x)$ for the following differential equations.

(a) $x^2y'' - 3xy' + 4y = 0$ with $y(1) = 4$ and $y'(1) = 5$ (15%)

(b) $y'' - y' - 2y = 10\cos x$ with $y(0) = 2$ and $y'(0) = 1$ (20%)

(c) $(2xy^4 + \sin y) dx + (4x^2y^3 + x\cos y) dy = 0$ (15%)

2. (10%) Derive the Laplace transform for the function $\{\sin kt\}$ is

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2 + k^2}$$

3. (20%) matrix $M = \begin{pmatrix} \alpha & 0 & 3\beta \\ 0 & \alpha & 4\beta \\ 3\beta & 4\beta & \alpha \end{pmatrix}$, the three eigenvalues of the matrix M are -1 ,

4, and 9. Please find the α and β [assume $\alpha > 0$ and $\beta > 0$]

4. (20%) Find the particular solution of following differential equation:

$$\begin{cases} \frac{dx}{dt} = 2x + y + 4e^{2t} \\ \frac{dy}{dt} = x + 2y \end{cases} \text{ which satisfies the initial condition } \begin{cases} x(0) = 4 \\ y(0) = 1 \end{cases}$$