

科目：工程數學

1. (50%) Solve $y(x)$ for the following differential equations.

(a) $x^2y'' - 3xy' + 4y = 0$ with $y(1) = 2$ and $y'(1) = 8$ (20%)

(b) $y'' - y' - 2y = 10\cos x$ with $y(0) = 0$ and $y'(0) = 2$ (20%)

(c) $y' = 1 + y^2$ (10%)

2. (10%) Derive the Laplace transform for the function $\{\sinh kt\}$ is

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2 - k^2}, \quad s > |k|$$

3. (20%) matrix $M = \begin{pmatrix} \alpha & \beta & 0 \\ \beta & \alpha & \beta \\ 0 & \beta & \alpha \end{pmatrix}$, the three eigenvalues of the matrix M are $-3, 1,$

and 5. Please find the α and β [assume $\alpha > 0$ and $\beta > 0$]

4. (20%) Please use Fourier series expanded $f(x) = x^2$ with $-\pi < x < \pi$ and show

$$\text{that. } \sum_{1}^{\infty} \frac{1}{s^2} = \frac{\pi^2}{6}$$

$$\text{Hint: } f(x) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos nx + \sum_{1}^{\infty} b_n \sin nx \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx; b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$\int x^2 \cos nx dx = \frac{x^2 \sin nx}{n} + \frac{2x \cos nx}{n^2} - \frac{2 \sin nx}{n^3} + C$$