

中國文化大學 103 學年度碩士班考試入學招生考試

系所組：化學工程與材料工程學系奈米材料碩士班

日期節次：103 年 3 月 15 日第 1 節 9:00~10:30

科目：工程數學

1. (50%) Solve  $y(x)$  for the following differential equations.

(a)  $y'' - y = 0$  with  $y(0) = 6$  and  $y'(0) = -2$  (15%)

(b)  $y'' + 4y = 2\sin 2x$  with  $y(0) = 1$  and  $y'(0) = 0.5$  (20%)

(c)  $xy^2 dx + (2 + x^2y) dy = 0$  with  $y(1) = 2$  (15%)

2. (10%) Derive the Laplace transform for the function  $\{\sin kt\}$  is

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2 + k^2}$$

3. (15%) matrix  $M = \begin{pmatrix} 5 & -5 & 0 \\ -1 & 2 & -1 \\ 0 & -5 & 5 \end{pmatrix}$ , Please find the three eigenvalues of the matrix M

4. (25%) Using the Fourier series to expand the following function:

(a)  $f(x) = \cos(ax)$ , with  $-\pi < x \leq \pi$  and  $a \neq \text{integer}$  (15%)

(b) if  $x = \pi$ , please show that:  $\cot(x) = \sum_{n=-\infty}^{\infty} \frac{1}{x + n\pi}$ ,  $n = \text{integer}$  (10%)

There are some useful formulae:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}, \text{ with } -L < x \leq L$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, n = 0, 1, 2, 3, \dots \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, n = 1, 2, 3, \dots$$

$$2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$

$$\sin(a \pm n)\pi = (-1)^n \sin a\pi, \text{ if } n = \text{integer}$$