

中國文化大學 102 學年度碩士班考試入學招生考試

系所組：化學工程與材料工程學系奈米材料碩士班

日期節次：102 年 3 月 15 日第 2 節 11:00~12:30

科目：工程數學

1. (50%) Solve $y(x)$ for the following differential equations.

(a) $y''+6y'+9y=0$ with $y(0)=2$ and $y'(0)=0$ (15%)

(b) $y''+y=x^2$ with $y(0)=0$ and $y'(0)=3$ (20%)

(c) $xy'+y^2=4$ (15%)

2. (10%) Derive the Laplace transform for the function $\{\cosh kt\}$ is

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2 - k^2}$$

3. (20%) matrix $M = \begin{pmatrix} \alpha & \beta & 0 \\ \beta & \alpha & \beta \\ 0 & \beta & \alpha \end{pmatrix}$, the three eigenvalues of the matrix M are $-3, 1,$ and 5. Please find the α and β [assume $\alpha > 0$ and $\beta > 0$]4. (20%) Please use Fourier series expanded $f(x) = x^4$ with $-\pi < x < \pi$ and show

that $\sum_1^{\infty} \frac{1}{s^4} = \frac{\pi^4}{90}$.

Hint: $f(x) = \frac{a_0}{2} + \sum_1^{\infty} a_n \cos nx + \sum_1^{\infty} b_n \sin nx$ $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$; $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$

$$\int x^4 \cos nx dx = \frac{x^4 \sin nx}{n} + \frac{4x^3 \cos nx}{n^2} - \frac{12x^2 \sin nx}{n^3} - \frac{24x \cos nx}{n^4} + \frac{24 \sin nx}{n^5} + C \text{ and}$$

$$\sum_1^{\infty} \frac{1}{s^2} = \frac{\pi^2}{6}$$