

1. (50%) Solve $y(x)$ for the following differential equations.

(a) $y''+y'+0.25y=0$ with $y(0)=3$ and $y'(0)=-3.5$ (15%)

(b) $y''+y=0.001x^2$ with $y(0)=0$ and $y'(0)=1.5$ (20%)

(c) $\cos(x+y) dx+(3y^2+2y+\cos(x+y)) dy=0$ (15%)

2. (10%) Derive the Laplace transform for the function $\{\cos kt\}$ is

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2 + k^2}$$

3. (20%) matrix $M = \begin{pmatrix} \alpha & 0 & 5\beta \\ 0 & \alpha & 12\beta \\ 5\beta & 12\beta & \alpha \end{pmatrix}$, the three eigenvalues of the matrix M are

-7, 6, and 19. Please find the α and β [assume $\alpha > 0$ and $\beta > 0$]

4. (20%) Find the particular solution of following differential equation:

$$\begin{cases} \frac{dx}{dt} = 3x + y + 3e^t \\ \frac{dy}{dt} = x + 3y \end{cases} \text{ which satisfies the initial condition } \begin{cases} x(0) = 5 \\ y(0) = 3 \end{cases}$$